School of Isolated and Distance Education
MATHEMATICS SPECIALIST Year 11

Test 2 2023

Section 2: Calculator assumed

Time allowed for this section

Working time: 35 minutes

Marks allocation: ~~38~~ 39 marks

PERMISSIBLE ITEMS

Standard Items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special Items: **Formulae Sheet, CAS calculator, ONE A4 page of notes**

STANDARD FORMULAE SHEET IS PROVIDED

NO OTHER ITEMS MAY BE TAKEN INTO THE EXAMINATION ROOM

INSTRUCTIONS FOR CANDIDATES

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

All work must be done in the space provided. Should you need extra working area you may use the blank pages at the end.

Student's name: Solutions

SIDE Teacher's name: _____

SUPERVISOR'S DECLARATION

I declare that this test paper has been completed without assistance by the student named above. The time and resource restrictions have been observed and the student has NOT accessed additional notes other than the one A4 page allowed, texts, reference books, the internet, a computer, a mobile phone or other electronic device. I understand that this paper will not be counted for assessment if these conditions have not been met and that notifications will occur.

Supervisor's name: _____

Supervisor's signature: _____ **Date:** _____

QUESTION 1 **[3 marks]**

Determine whether the statement below is true or false. If it is false, find a counterexample.

$$If\ a > b, then\ \frac{1}{a} < \frac{1}{b}, \quad a \text{ and } b \neq 0$$

$a = 4$, $b = 3$ ✓

but $\frac{1}{4} \not< \frac{1}{3}$ ✓

False ✓

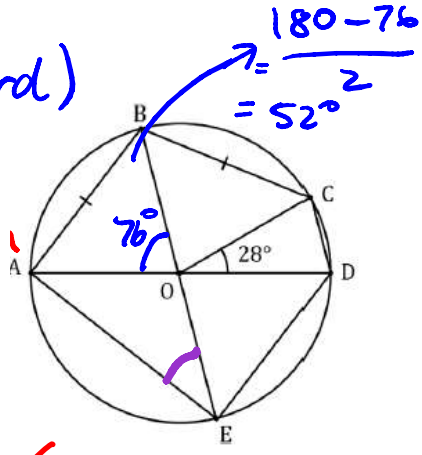
Supply example
Check example.
a and $b \neq 0$ ✓

3

QUESTION 2 **[1, 1, 1 = 3 marks]**

In the diagram below, AD and BE are diameters of the circle with centre O, C lies on the circumference and $\angle COD = 28^o$.

Determine the sizes of the following angles, give reason for each of your answer.



(a) ∠AOB

$\angle AOB = \angle BOC$ (equal chord)

$\therefore \angle AOB = \frac{180 - 28}{2}$

$= 76^o$ ✓

$= \frac{180-76}{2}$

$= 52^o$

(b) ∠AEB

$\angle EAB = 90$ (angle in semi circle)

$\angle BAE = 90 - 52 = 38^o$ ✓

(c) ∠EAB

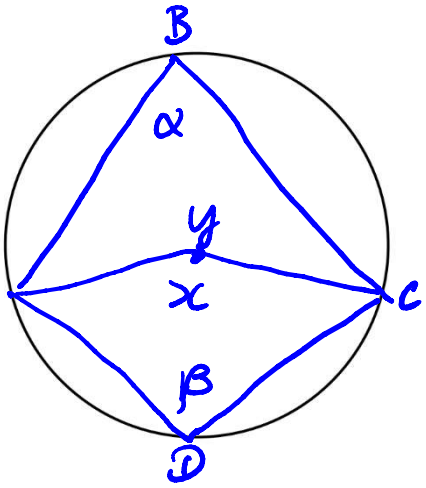
$\angle EAB = 90^o$ (angle in semi circle) ✓

−1 mark for "no reason" for the whole of this question.

QUESTION 3 **[4, 4, 2 = 10 marks]**

(a) If a quadrilateral is a cyclic quadrilateral, prove that each pair of opposite angles sum to 180°.

RTP: each pair of opposite angle sums is 180°



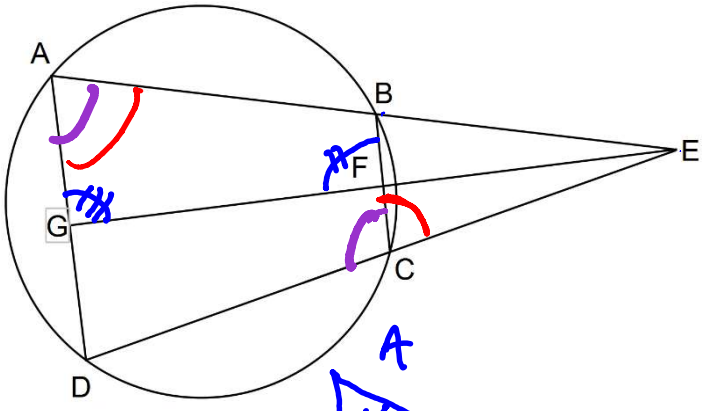
$\begin{cases} x = 2\alpha \\ y = 2\beta \end{cases}$ (Angle in centre theorem) ✓ Reason

$2\alpha + 2\beta = 360$

$\alpha + \beta = 180$ ✓

∴ The sum of opposite angles in cyclic quadrilateral is 180° ✓ Conclusion (4)

(b) ABCD is a cyclic quadrilateral. Bisector of ∠BEC intersects BC at F and AD at G.
Prove: ∠AGF = ∠BFG



∠EAG + ∠DCB = 180 (ABCD cyclic quad) ✓

∴ ∠EAG = ∠ECF

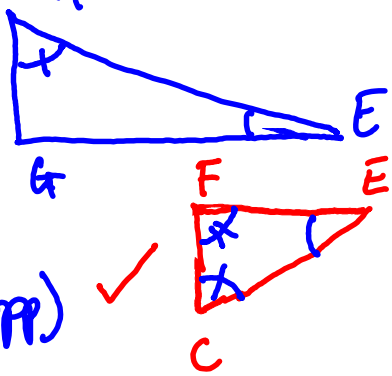
In ΔEAG and ΔECF

∠AEG = ∠CEF (bisector) ✓

∠EAG = ∠ECF (proven)

∴ ∠CFE = ∠BFG (Vertically opp) ✓

∴ ∠AGF = ∠BFG ✓



(QUESTION 3 continued)

Consider the statement:
"If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic."

(c) (i) In relation to the theorem in (a), this statement is which of the following? converse, negation, inverse or contrapositive

Converse ✓

(2)

(ii) Is the statement true? yes ✓

QUESTION 4 [4 marks]

Find the values of the pronumerals for the following:

$y = x$ (angle in the same segment) ✓

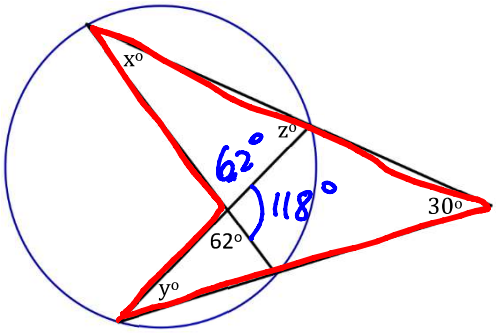
$2x + 30 + 242 = 360$ ✓

$2x = 88$

$x = y = 44°$ ✓

$z = 180 - 44 - 62$

$= 74°$ ✓



(4)

✓ Reason for $x = y$

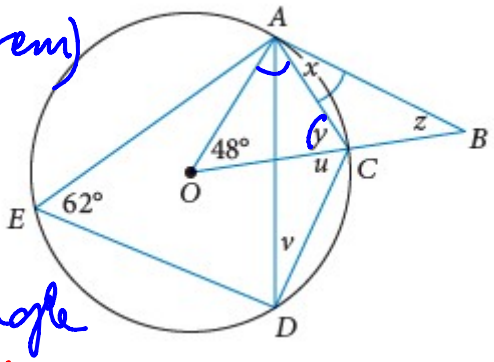
✓ correct z

✓ Correct x

✓ Working out

QUESTION 5 **[2, 2, 2, 2, 2 = 10 marks]**

If O is the centre of the circle, AB is a tangent, determine the value of the following pronumerals and give reason for each step of your working out.



(a) v

$2v = 48$ (angle in centre theorem) ✓

$v = 24°$ ✓

(b) y

AOC is isosceles triangle ✓

$y = \frac{180 - 48}{2} = 66°$ ✓

(c) u

$y + u + 62° = 180°$ (opposite angles in cyclic quad) ✓

$\therefore u = 52°$ ✓

(d) x

$\angle OAB = 90°$ (angle b/t radius and tangent is 90°) ✓

$x + 66 = 90$

$x = 24°$ ✓

(e) z

$\triangle OAB$ ✓

$48° + z = 90$ (Angles in triangle add to 180°)

$\therefore z = 42°$ ✓

✓ mark for correct answer

✓ mark for correct reason

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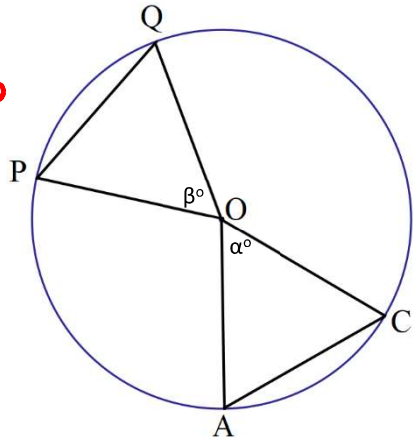
6 3 9

QUESTION 6 **[3, 5 = 8 marks]**

AC and PQ are chords of the circle centre O.



(a) Prove that $\alpha = \beta \iff AC = PQ$

If $\alpha = \beta$ ✓ Need to state this

$\Delta POQ \equiv \Delta AOC$ (SAS)

$OA = OC = OP = OQ$ (radius)] ✓

$\angle POQ = \angle AOC$

$\Rightarrow PQ = AC$ ✓

If $AC = PQ$ ✓ Need to state this

$\Delta POQ = \Delta AOC$ (SSS)

$OA = OC = OP = OQ$ (radius)] ✓ for proving

$\angle AOC = \angle POQ$

$\alpha = \beta$

$\therefore$ If $AC = PQ$ then $\alpha = \beta$. ✓ conclusion

(b) In addition to $\alpha = \beta$, if P, O and C are collinear (three points lie on a straight line) and Q, O and A are collinear, prove that PQ parallel to AC.

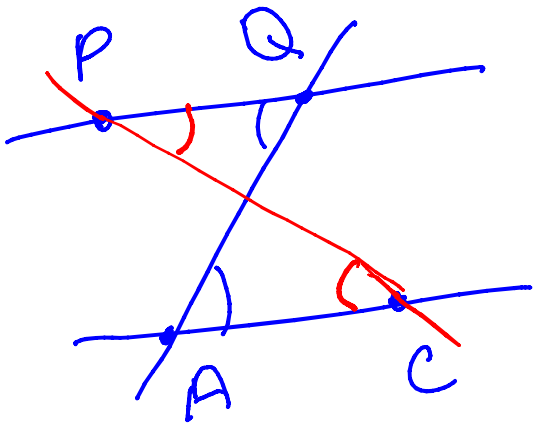
Since $\alpha = \beta$ $\Delta POQ \equiv \Delta AOC$

$\angle OPQ = \angle PQO = \angle OAC = \angle OCA$ ✓

Since P, O, C and A, OB are collinear

and $\angle OCA = \angle OPQ$ ✓

$\Rightarrow PQ \parallel AC$ → Alternate angles. ✓



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End of Test

Additional page for working out